ON PROPORTION/RATIO

The historian's preoccupation with the literary and mathematical remains of proportional theory, particularly with the suggestive but imperfectly documented traditions of the Pythagoreans, can easily overlook a fundamental point: that the origins of proportionate art ... are much older than philosophy and mathematics – older, probably, than writing itself. In the Egyptian hieroglyph for 'town', a circle enclosing a centrally located cross, we already find the essential geometry of urban form concisely expressed in the symbol or *mandala* which traditionally encoded the analogy of the macrocosm and the microcosm, and the idea of the city or temple as an *imago mundi*. The cosmic associations of the circle (symbolising heaven) and the square (symbolising earth) are found in all the old Eurasian civilisations, which must therefore be presumed to have shared this formative world-view. The high antiquity of these ideas is confirmed by the etymology of key technical terms, some of which take us back to the oldest root-stock of the Indo-European languages. For example, the root, (h)ar, is common to the Greek *harmonia*, the Sanskrit *rta* (world order) and the Latin *ritus* (whence the English terms, rite and ritual): these and many other related terms convey the notion of fitting together opposites, at all cosmic levels, into a balanced whole. The same world-view connects the root, tem, with *temenos* (the sacred circular space), *templum* (the divine house within the sacred space), *tempus* (perhaps expressing the image of time as circular) and apparently also *template* (which preserves the idea of an outline or analogical form). Closely related is the extensive vocabulary of tuning, words such as temper, temperance, temperament, which, as Leo Spitzer has shown, is employed throughout the cosmic hierarchy and widely diffused among the European languages. The distribution of this terminology alone demonstrates the cultural significance of ... proportion, especially *due proportion*, a concept deployed in most areas of human regulation and control, including ethics, law, politics, medicine, architecture and the polite and ornamental arts generally. The unifying principle here is music, whether in the concrete forms of human song and dance or in the more intellectual abstractions and cosmic associations of the vibrating string. ...[6]

The proportional 'fitting together' of complementarities into a holistic system is a *harmonia* that begets beauty, and is attained through creating ratio (or *logos*) via number and geometry. The unity behind multiplicity in such a cosmos has ratio as its binding mechanism; proportion generates this interrelational unity and is also a means of analysing it. (Indeed, unity revels in differences – an inspiring paradox!):

According to the Pythagoreans, the cosmos is made up of a dynamic harmony of opposites or complementary forces. In the same way that the year encompasses both winter and summer, night and day, so too is the universe a living harmony of opposing tendencies and forces. ... by applying the word *kosmos* to the universe, Pythagoras was saying that the earth and the heavens are adorned and ornamented by beauty. It implies that there is an order to the universe ... Pythagoras said that [our universe, the] *kosmos* is a *harmonia*, a "fitting together". Developed in a mathematical sense by the Pythagoreans, the root concept of *harmonia* is truly primordial; the term *harmony* springs from the prehistoric (*circa* 5000 BC) Indo-European root **ar-**, "to fit together", which is the root of the words ARM, HARMONY, ART, ORDER, ORNAMENT, ADORN, RATIO, REASON, READ, RITE, ARITHMOS (number), and RHYME. Through the principle of harmony, the parts fit together into the whole. Even though the universe contains an unlimited number of things, phenomena, and forces, they are all miraculously and beautifully reconciled into the greater whole. This is the law of "unity in multiplicity" which fascinates every careful observer of the natural world. In their philosophy, the Pythagoreans expressed these relationships through the natural language of number and geometry. Number is a natural language

because, like the harmonic ratios in music, it is not invented, but discovered. However, the Pythagorean vision of Number is quite different from the way we imagine it today. While we use numbers to count things in a quantitative sense, the Pythagoreans saw Number as a *qualitative* essence, a principle of relationship or *logos*. ...[12]

As the Pythagoreans realized, the study of proportion and harmony constitutes an objective science and comprises the starting-point for a philosophy of whole systems. A philosophy of whole systems is based on the premise that there is an underlying unity behind the nature of things, and that the many parts of an organism are intrinsically related to one another within the context of a greater whole. This principle of relatedness is seen to exist inherently in the structure of the human body, the biosphere, and the structure of the solar system. A philosophy of whole systems recognizes this "fitting together" [*harmonia*] and trains the mind to see how the parts relate to the whole; it helps us to think in terms of whole systems, leading our minds and our conceptions to follow the path of Nature itself. In order to achieve this type of "proportional thinking", the ancient Pythagoreans both studied and exposed their intellects to the pure principles of geometrical and musical harmony, which are the most primeval types of natural relationship (*logos*), and thus underlie the more complex organic relatedness of phenomena in the natural world. A philosophy of whole systems starts with the realization that there is unity behind diversity, and that, in the manifest world, unity expresses itself through the differentiated image of multiplicity. This manifestation is controlled by the related principles of Number, Logos, and Harmony, as seen in the Pythagorean sense. ...[13]

"LINES OF LIGHT" AND FRAUNHOFER LINES AS TEMPORAL DETERMINANTS

Name	Wavelengths *	Colours †	Elements ‡	With H = 0	Differences *
A	≈ 7594 Å	Red	O_2 {Oxygen}	3626	A – C = 1031
C	≈ 6563 Å	Red-Orange	H {Hydrogen}	2595	C – D = 670
D	≈ 5893 Å	Yellow	Na {Sodium}	1925	D – E = 623
E	≈ 5270 Å	Green	Fe {Iron}	1302	E – F = 409
F	≈ 4861 Å	Blue	H {Hydrogen}	893	F – G = 553
G	≈ 4308 Å	Violet	Fe & Ca {Iron & Calcium}	340	G – H = 340
H	≈ 3968 Å	Ultraviolet	Ca {Calcium}	0	—

* The Fraunhofer lines' wavelengths are measured in Angstrom units (Å): 1 Å = 10^{-10} metres. NB: because the data in the 'differences' column will eventually be reduced to proportions (thence to relative durations), the actual unit of measurement used here becomes irrelevant.

Astonishingly, those three aeonial constants $\sqrt{2}$, $\sqrt{3}$, and $\phi = ((\sqrt{5} - 1) \div 2)$ are also latent herein, linking (albeit approximately) the wavelength of Fraunhofer line E to that of Fraunhofer line G – i.e. 5270 Å × $\sqrt{2}$ ÷ $\sqrt{3}$ ≈ 4302.94 Å ≈ 4308 Å – as well as mediating the two wavelength-intervals between Fraunhofer lines F, G and H – i.e. 553 Å × ϕ ≈ 341.77 Å ≈ 340 Å.

† The Fraunhofer lines lie within these colours of the rainbow.

‡ Each Fraunhofer line correlates with a spectral line (or narrow band) belonging to the indicated element.

The atomic-spectral homologies in this column have been corroborated also by a reliable Internet website, Harmsworth (2001).
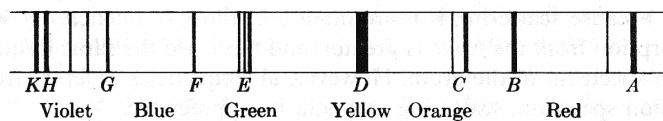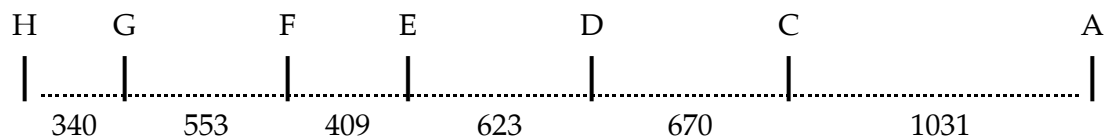
H	G		F	E		D		C		A
	340		553		409		623		670	1031

Figure 30.5 The Fraunhofer dark lines in the visible part of the solar spectrum. Only a few of the most prominent lines are represented here.

ON THE SYMBOLISM OF 'RAINBOWS' AND OF 'LIGHT' IN GENERAL

Beyond that well-known story told in Genesis 9:8–17, of the rainbow as symbol of a covenant between Noah (i.e. humankind) and God, the rainbow itself is rich in imagery:

The rainbow is frequently a symbol of the union of heaven ... and earth. According to Talmudic tradition, the rainbow was created on the evening of the sixth day of Creation. In Greek mythology, the rainbow is the embodiment of the messenger of the goddess Iris; in Germanic mythology, it is the bridge Bifröst joining Asgard and Mid-gard. After the Flood, God placed a rainbow in the sky as a sign of His covenant with humans; in medieval depictions of Christ as ruler of the world, for example, Christ reigns on a rainbow, which is to be understood in this sense. Thus, the rainbow also became a symbol of Mary, the intermediary of reconciliation. The symbolic interpretation of the colours of the rainbow depends on how many colors one distinguishes; in China, for example, five colors of the rainbow are recognized, their synthesis symbolizing the union of Yin and Yang. In accordance with the Aristotelian tradition of a three-fold division, only the three primary colors (a symbol of the Trinity) are distinguished in Christianity; yet the colors blue (the water of the Flood or the heavenly origin of Christ), red (the coming destruction of the world by fire or the Passion of Christ), and green (the new world or Christ's workings on earth) are also distinguished. ...

The rainbow, a striking and central symbol and sign, articulates man's hope for a better world. Is it any wonder that the rainbow, one of the greatest symbols, became the one that was most misused, robbed of its meaning, and commercialized? If one pursues the history of the rainbow symbol back to the beginnings of literary tradition, then we find the rainbow in Babylonian literature as a sign of wrath, of terror, and of ill, and as an attribute of the goddess Tir-an-na, bringer of misfortune. This aspect also appears in ancient Jewish writings. Specifically, an apocryphal writing claims that God removed the layer of wrath from the bow in the clouds, meaning that He detached the band of fury from the devastating waters of the Flood, which came from the high windows of heaven, and then spread out the bow in the clouds: instead of the flood waters, the rainbow, created on the eve of the first sabbath, now shines as the arch of peace. Thus, the rainbow underwent a complete change in meaning: the bow went from being a symbol of wrath and terror of the goddess Tir-an-na to the symbol of peace, the sign of the covenant between Yahweh and man. In this context, Alfons Rosenberg has an interesting idea. In particular, he suspects that the bow mentioned in Genesis 9:8–17 does not at all refer to a rainbow, but rather to the arc of the zodiac. This would indeed be a much deeper embedment of the mundane event of the Flood in a larger, more comprehensive cosmic context than would be possible with the rainbow image.[3]

Concerning the symbolism of 'light' in general, Udo Becker points out that it is an "omnipresent phenomenon that is well known to us in its effects but whose essence is largely unintelligible. It is thus a favored symbol of immateriality, spirit, and God as well as [of] life or happiness. One occasionally encounters a finer distinction between the light of the Sun, which symbolizes inspiration and spiritual vision, and the light of the Moon, which, as reflected light, symbolizes mediate forms of knowledge through rational, discursive thought. Light is frequently encountered as a border of darkness, which then usually appears as a symbol of ignorance and spiritual dullness, of morally underdeveloped or inferior areas and conditions, of death, misfortune or also of "mystery". In symbolic thought, the spatial notion of "up" and "down" ... corresponds to the relation between light and darkness. Nearly all fundamental principles based on a division of the world into a duality, refer to this distinction of light and darkness, such as Ormuzd and Ahriman [in Persian thought], Yin and Yang, angels and demons, spirit and

matter, masculine and feminine. The idea of an ascent from darkness to light plays an important role for many peoples with respect to the development of humanity as well as to that of the individual; numerous initiation rites are thus based on this duality. The separation of light and darkness as a postulate of the initial order at the beginning of the world is encountered in the cosmogonies of many peoples. Mystics sometimes speak of a darkness that lies "beyond" (rather than "beneath") the light of knowledge and that symbolizes the essential incomprehensibility of God. In the visual arts, the spiritual enlightenment of a person is frequently shown by an aureole, a nimbus, or a halo. ..."

"DIMENSIONES PARADISI" AND THE 'NEW JERUSALEM' MANDALA

My abbreviations for the foundational "New Jerusalem" patterns in **Dimensiones Paradisi** are:

12-gon — a dodecagon (which is not quite perfectly regular);

C_1, C_2, C_3, C_4, C_5, C_6 — six concentric circles;

S — a square;

Δ — an equilateral triangle;

HS — a 'hexagonal star' (or hexagram), i.e. the six-pointed Star of David;

OH — its 'outer hex', i.e. a regular hexagon.

Those "New Jerusalem" constants that permeate **Dimensiones Paradisi** are:

Pattern	Section	Duration-Constants (ratios of AREAS)	Gamut-Constants (ratios of PERIMETERS)	Gamut-Spans (to the nearest eighthtone)
12-gon	G	$D_1 \approx 1.018498438$	$\Gamma_1 \approx 1.021235027$	no gamuts in sections G and H *
C_5	A, A'	$D_2 = 1.000000000$	$\Gamma_2 = 1.000000000$	132 eighthtones $= \Gamma_2 \times 132$
C_1	B, B'	$D_3 \approx 0.677847709$	$\Gamma_3 \approx 0.823315073$	109 eighthtones $\approx \Gamma_3 \times 132$
S		$D_4 \approx 0.532380346$	$\Gamma_4 = \Gamma_3$	109 eighthtones $\approx \Gamma_4 \times 132$
C_2	C, C'	$D_5 \approx 0.418130546$	$\Gamma_5 \approx 0.646630146$	85 eighthtones $\approx \Gamma_5 \times 132$
OH		$D_6 \approx 0.345791178$	$\Gamma_6 \approx 0.617486304$	82 eighthtones $\approx \Gamma_6 \times 132$
HS		$D_7 \approx 0.230527452$	$\Gamma_7 \approx 0.713011768$	94 eighthtones $\approx \Gamma_7 \times 132$
Δ		$D_8 \approx 0.172895589$	$\Gamma_8 \approx 0.534758826$	71 eighthtones $\approx \Gamma_8 \times 132$
C_3	D, D'	$D_9 \approx 0.104532636$	$\Gamma_9 \approx 0.323315073$	43 eighthtones $\approx \Gamma_9 \times 132$
C_4	E, E'	$D_{10} \approx 3.121756336 \times 10^{-2}$	$\Gamma_{10} \approx 0.176684926$	23 eighthtones $\approx \Gamma_{10} \times 132$
C_6	F	$D_{11} \approx 1.382249740 \times 10^{-4}$	$\Gamma_{11} \approx 1.175691175 \times 10^{-2}$	2 eighthtones $\approx \Gamma_{11} \times 132$

* NB: section H falls completely outside my sacred-proportionalized framework of pitch-gamuts and durations, being composed 'non-systemically' at a later date than sections A to G and E' to A'.

CONCERNING OMPHALOI IN GENERAL AND THAT OF THE 'NEW JERUSALEM' MANDALA IN PARTICULAR

The formal design of **Dimensiones Paradisi** stems from "a complex mandala-like figure incorporating a dodecagon, several concentric circles, a hexagon, and the Star of David"[18] whose central point, or *omphalos*,[19] is

[in] the myths of various peoples, ... a symbol of the center of the world from which creation is supposed to have originated. The omphalos of Delphi is famous; it is a cylindrical stone with a rounded top that was also a symbol of the connection between the realms of the gods, of man, and of the dead. The pole star is sometimes thought of as the navel of the sky around which the heavens seem to rotate.[20]

Raphael Patai further explains the significance of the *Omphalos* within Jewish mythology:

In the middle of the Temple [of Jerusalem], and constituting the floor of the Holy of Holies, was a huge native rock which was adorned by Jewish legends with the peculiar features of an *Omphalos*, a Navel of the Earth. This rock, called in Hebrew *Ebhen Shetiyyah*, the Stone of Foundation, was the first solid thing created, and was placed by God amidst the as yet boundless fluid of the primeval waters. Legend has it that just as the body of an embryo is built up in its mother's womb from its navel, so God built up the earth concentrically around this stone, the Navel of the Earth. And just as the body of the embryo receives its nourishment from the navel, so the whole earth too receives the waters that nourish it from this Navel.[23]

THE TRIANGLE: ITS SYMBOLISM, AND THAT OF ITS ASSOCIATED NUMBERS 3, 9 AND 27

The triangle's profile automatically invokes the numbers '3', '9' (being 3×3) and '27' (being $3 \times 3 \times 3 = 3^3$), as well as the Holy Trinity:

[3 is the] basic number of the masculine principle next to 1 (the number of the divine) and 2 (the number of the feminine); water is the [alchemical] element of 3; the triangle is its coordinate geometrical figure. The triangle and water are also associated with one another in alchemist-medical signs. 3 plays an unmistakably fundamental role in all religions. The second and third powers of three [i.e. 9 and 27] are of particular significance, ... [being] regarded as a "strengthening" of the symbolic power of 3. 3 is the basis of numerous systems and ordering schemes; thus, Christianity, for example, has the 3 virtues of faith, love, and hope, and alchemy has the 3 basic principles of sulphur, salt and quicksilver ... Divine triumvirates are known in many religions, for example in Egypt (Isis, Osiris, Horus), in Hinduism (Brahma, Vishnu, Shiva), etc. Such divine triumvirates often appear in conjunction with heaven, earth and air (which binds them together). In contrast to this, Christianity has a triune God who is often envisaged as being a unity of three persons (a trinity). As the number of fulfillment of a self-contained entirety, 3 is frequently encountered in fairy tales as the number of tests that one must withstand or riddles that one must solve, etc. In philosophy, the triad or triple-step plays an important role, for example, as the principle of mediation between thinking and being or, as in the case of Hegel, as the principle of dialectical progress (thesis, antithesis, synthesis). ...

The number 9 is the second power of 3 [i.e. 3^2] ... the amplification of the sacred 3; this is the reason why the Kyrie eleison in the Roman Catholic liturgy is repeated nine times and why there are 9 choirs of blessed spirits; 9 plays an important role in Indo-Germanic and Central Asian mythology as well, as in the case of the 9-storied pagoda, which is a symbol of heaven.[14]

As the third power of 3, i.e., $3^3 = 27$, [the number 27] is the amplification and perfection of 3; it is the number of the element fire ... Under favorable conditions, the Moon is visible for $3 \times 9 = 3 \times 3 \times 3$ nights.[15]

On the symbolism of the triangular form itself, Udo Becker asserts that:

To a large extent, the triangle shares in the symbolic significance of the number three. In antiquity, it was sometimes regarded as a symbol of light. For many peoples, it is a symbol of fire and masculine virility when its tip points upward and a symbol of water and the female sex when its tip points downward. The equilateral triangle is often used as a sign for God or for harmony. In Christianity, it is a symbol of the Trinity (especially since the 17th century, it frequently appears in conjunction with a hand, a head, an eye, or the Hebrew name of God, Yahweh). According to popular custom, magicians and sorcerers, the triangle is an apotropaic sign. For the Freemasons, the triangle plays an important role as (among other things) a symbol of God's strength, beauty and wisdom; of the cornerstone of the Freemasons' temple; of the mineral, plant and animal kingdoms; of the three stages of a person's spiritual development (*Separatio*, *Fermentatio* and *Putrefactio*); of correct speech, thought, and action; of birth, maturity and death; and so forth.[16]

THE SQUARE: ITS SYMBOLISM, AND THAT OF ITS ASSOCIATED NUMBERS 4 & 16

Concerning the imagery of the square form – and of its correlative number '4', as well as 4's second power '16' – the square is one of

the most common symbolic signs; it is a static, non-dynamic symbol, often seen in relation and in contrast to the circle, a symbol of the earth in contrast to heaven ... or of the limited in contrast to the unlimited. It is also a symbol of the four cardinal directions [north, south, east, and west]. It is frequently used as a foundation of temples, altars, cities or as an architectonic unit, such as in the Romanesque style. In China, the cosmos and the earth were thought to be quadratic. The Pythagoreans saw the square as a symbol of the united workings of the four elements ['air', 'water', 'fire', and 'earth'] and thus of the powers of Aphrodite, Demeter, Hestia and Hera, the synthesis of whom was thought to be Rhea, mother of the gods. According to Plato, the square, next to the circle, embodies absolute beauty. In Islam, the square plays a role in various contexts; the hearts of normal people, for example, were thought to be square, because they were open to four possible sources of inspiration: the divine, the angelic, the human, and the devilish (the hearts of the prophets, on the other hand, are triangular, because they are no longer subject to the devil's attacks). In Christian art, the square is sometimes a symbol of the earth in contrast to heaven. Square haloes of people (then) still living thus indicate that the figure is still of this earth. C. G. Jung sees the square as a symbol of matter, of the flesh, and of earthly reality. [On the quadrature of the circle, it is the] unsolvable task of converting a given circle into a square of equal area by using only a ruler and compass. It is a symbol of the attempted [inter]penetration of the symbolic meaning[s] of the circle and the square.

[Four is the] basic number of the feminine, also regarded as a cosmic number and a number of harmony; it is based on the second power of 2, i.e., $2^2 = 2 \times 2 = 4$; with the 4 seasons, it is a manifestation of Mother Earth = 4; the number 2, with its maternal principle, thus expands to include the cosmos, which is bounded by 4 [NB: Albert Einstein saw our Universe, physically, as a four-dimensional space-time continuum]. As a symbolic number, 4 is closely related to the square and the circle. It is the number of the 4 cardinal directions and thus of the 4 winds, the 4 [alchemical] elements ..., the 4 humors, the 4 rivers of Eden, the 4 Evangelists, and the 4 stages of life (childhood, youth, maturity, old age).

The number 16 is a number symbolic of completeness and perfection and is probably associated with the four elements in their squared form, i.e., $4^2 = 4 \times 4 = 16$.[9]

The Circle: its symbolism

The circle itself is

[o]ne of the most common signs, often seen in relation and contrast to the square. The circle leads back into itself and is thus a symbol of unity, of the absolute and of perfection; it is thus also a symbol of heaven in contrast to earth or of the spiritual in contrast to the material; there is a close association to the symbolic significance of the wheel. As an infinite line, it is a symbol of time and infinity, often symbolized by the figure of a serpent biting its own tail. For practitioners of magic, the circle is an effective symbol of protection against evil spirits, demons, etc.; this is probably the reason for the protective function attributed to the belt, the ring, the hoop, the circular amulet, etc. In Zen Buddhism, concentric circles symbolize the highest level of enlightenment and the harmony of all spiritual powers; in other contexts, such as in Christianity, they symbolize various spiritual hierarchies or the various levels of creation. In Christianity, three intersecting circles symbolize the Trinity. The circle inscribed in a square [a geometric conjunction which is featured within the "New Jerusalem" diagram] is a common Cabalist symbol for the spark of divine fire lying hidden within matter. C. G. Jung sees the circle as a symbol of the psyche and of the self.[7]

ON THE QUADRATURE OF THE CIRCLE, THE SYMBOLIC RESOLUTION OF OPPOSITES

The sacred mandala-like "New Jerusalem" pattern (as depicted by John Michell in his book **The Dimensions of Paradise: The Proportions and Symbolic Numbers of Ancient Cosmology**)[4] acts both as an archetypal ground-plan for many edifices and, in particular, as an entelechial 'force-field' that permeates **Dimensiones Paradisi** – one which regulates this piece's pitch-gamuts and time-spans.

A significant underlying principle behind Michell's "New Jerusalem" pattern can be summarized as *the symbolic resolution of antitheses*, represented chiefly therein by *the quadrature of the circle* – i.e. a circle and a square interpenetrate one another and harmonize, such that they both accommodate either identical areas or commensurate perimeters.[6]

> David R. Fideler, in his Introductory Note to Michell (1991), p.72, elaborates: "The ancient Greeks were obsessed with the notion of the 'quadrature of the circle', or how it might be possible to geometrically construct a square and a circle of equal perimeter. Technically speaking, this is an impossible construction, owing to the irrational nature of pi. Due to this very fact, however, the problem of 'squaring the circle' possesses its intrinsic, symbolic appeal, for the reconciliation of the transcendental, 'heavenly' circle with the perfectly rational 'terrestrial' square represents the living fusion and reconciliation of opposites: heaven and earth, male and female, spirit and matter. Consequently, the geometry of the 'circle squared' has been repeatedly used in temple architecture over the ages, seeing that the temple itself represents a meeting place between heaven and earth, the eternal and temporal realms of existence. Despite the fact that the quadrature of the circle is technically impossible, there are a good many ways that the quadrature can be very accurately approximated in practice with only a compass and straight-edge. …"

THE HEXAGRAM: ITS SYMBOLISM, AND THAT OF ITS ASSOCIATED NUMBER 6

Another prominent unifying shape within the "New Jerusalem" mandala is the hexagram (and, by extension, the hexagon – formed through the joining of consecutive vertices of the hexagram with straight lines).

The hexagram is synonymous with

the "Seal of Solomon" and the "Star of David". [It is a] six-pointed star, formed by two triangles lying on or intertwined with one another; [the hexagram is] found particularly in Judaism, Christianity and Islam, yet is essentially the basis of the Indian *yantra* as well. In the broadest sense, the hexagram is often a symbol of the interpenetration of the visible and invisible worlds; in Hinduism, it is a symbol of the joining of *yoni* and *linga* [i.e. feminine with masculine]; in alchemy, it is also a symbol of the union of all opposites, since it is composed of the basic shapes of the signs of the elements fire Δ or air ⌂ and water ∇ or earth ⩢. One can also find numerous other speculations in alchemy that assume a correspondence between the individual lines or points of the hexagram and planets, metals, and qualities. The Star of David is a symbol of faith in Judaism and is the national emblem of the state of Israel. C. G. Jung sees the hexagram as symbolizing the unification of the realms of the personal and the impersonal or also of the masculine and the feminine.[8,6b]

The overlay or intermeshing of two triangles in the Star of David emblem is likewise a symbol of union, integrity, and wholeness (e.g. amalgamating masculine and feminine principles); this 'binding of opposites' is confirmed by Becker (1996), pp.308–309 – "For many peoples, [the triangle] is a symbol of fire and masculine virility when its tip points upward and a symbol of water and the female sex when its tip points downward" – and more expansively by Fideler (1993), pp.210–211:

KOSMOS: THE ORDERED WORLD

∇	[an encircled Star of David]	Δ
water	reconciliation	fire
instinct	of opposites	intellect
luna		sol
involution	ΚΟΣΜΟΣ	evolution
passive	cosmos = 600	active

"In the emblematic, alchemical language of traditional cosmology, the six-pointed star symbolizes the marriage of opposites through a third and higher medium, representing the creative union of the analytical intellect with the mythopoeic matrix of imagination. The upright triangle, as a symbol of the solar ego, when balanced with the inverted triangle of the lunar unconscious, produces the birth of the philosophical son: the heart and higher self. The Pythagorean philosopher Nicomachus [of Gerasa] called the hexad "The form of forms, the only number adapted to the soul . . . and the producing cause of the vital habit. Hence also, it is harmony, the perfection of parts, and more properly Venus herself." The archetypal geometry of the hexad, as an underlying design type of both natural phenomena and the three-dimensional [orthogonal] coordinate system itself, holds a special place in the systems of traditional cosmology. It is the geometrical basis of the Pythagorean Tetraktys, the kabbalistic Tree of Life, and the "fish net" symbolism of the early Christian gnostics. As a prime archetype underlying three-dimensional space and the worlds of inner and outer phenomena, there is every reason to believe that this matrix of creation will eternally influence the fabric of nature. ..."

Much of the iconology of the hexagram's cognate number, six, is also noteworthy; :

In antiquity and during the Middle Ages, the number 6 was regarded as the most perfect number because it can be represented as the sum of its parts and as their product, i.e., 1 + 2 + 3 = 6 and 1 × 2 × 3 = 6. It can also be thought of as the product of the first masculine number and the first feminine one, i.e., 3 × 2 = 6. In China, 6 was associated with the influences of the sky. In Christian symbolic thought, 6 is ambivalent: as the number of the days of creation, it is holy; it is also significant as the number of works of charity; in the Book of Revelation, however, 6 appears as the number of evil; 666 is the number of the beast of the apocalypse. ...[9]

Being a close geometric relative, the hexagon surely shares many of these symbolic meanings – particularly those associated with the number six.

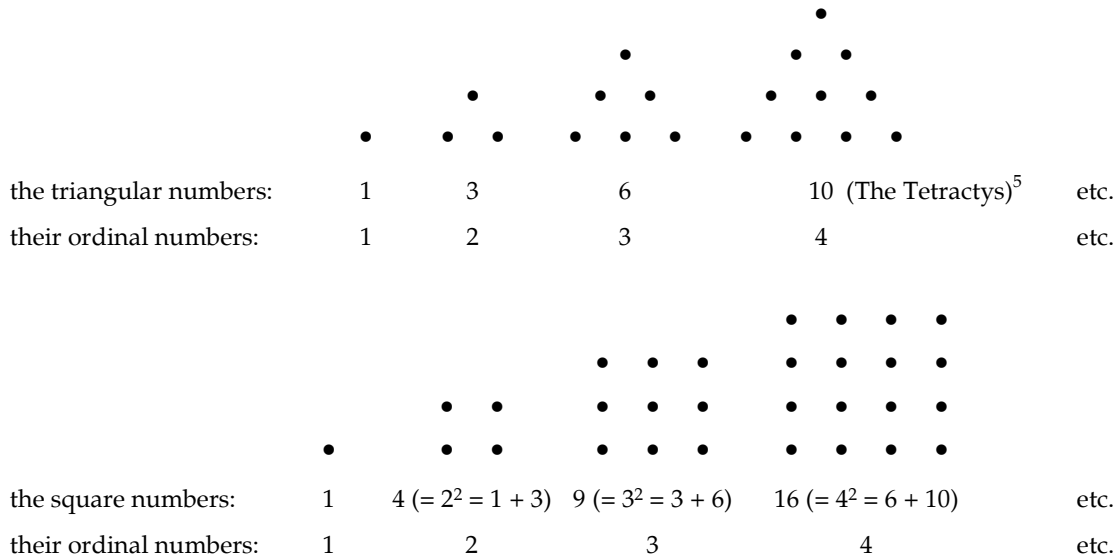## FIGURATE NUMBERS AND 'POLYGONAL DURATION-SETS'

Historically, *figurate numbers* are the outgrowth of an ancient accounting method which – even in our own 'information era', dominated by the digital computer – one can still glimpse within the etymology of the word *calculation*:

> ... it is interesting to see again how a residue of these things remains in some of our modern words. The use of stones for tallying is seen in the root of our word 'calculate' which derives from the Latin *calculus* meaning a 'pebble'. Even in comparatively modern times one finds the very simple use of stones to enumerate large collections.[2]

Greek arithmeticians compiled figurate numbers additively, by accumulating small objects (such as pebbles) into ever larger, orderly geometrical shapes:

> The[se] numbers are called *figurate* because early mathematicians perceived that they give the areas and volumes of certain geometrical figures when built up by discrete units. For example, if you place a number of [identical] cannon balls in the form of an equilateral triangle, with *n* balls in the base, then there will be *n–1* [balls] in the row above it, *n–2* [balls] in the row above that, and so on until you reach the apex of one ball. The total number of balls in the triangle will be the sum of the integers from 1 up to *n*. All such numbers are ... consequently called *triangular numbers*.[3]

Let us now 'look at' the first few *triangular numbers* and *square numbers*, here demonstrated as triangular and square collocations of dots:

the triangular numbers:	1	3	6	10 (The Tetractys)[5] etc.
their ordinal numbers:	1	2	3	4 etc.

the square numbers:	1	4 $(= 2^2 = 1 + 3)$	9 $(= 3^2 = 3 + 6)$	16 $(= 4^2 = 6 + 10)$ etc.
their ordinal numbers:	1	2	3	4 etc.

'Polygonal durations' derived from 'figurate numbers', when taken collectively, are a clear signifier of the noumenal geometric 'figure' – a polygon which, in turn, will register various cultural associations (as with a mandala):

Layer	Polygon Type	a_k (in order)											A_k	'Tempo Ratios'
1	triangular	1	2	3	4	5	6	7	8	9	10	11	$A_{11} = 66$	66:66 = 1:1
2	square	17	15	13	11	9	7	5	3	1			$A_9 = 81$	81:66 = 27:22
3	*centred* pentagonal	30	25	10	15	1	5	20					$A_7 = 106$	106:66 = 53:33
4	triangular	1	2	3	4	5	6	7	8	9	10	11 12	$A_{12} = 78$	78:66 = 13:11

EXERCISES: CREATING ART-WORKS USING THESE CONCEPTS

A. SOME BASIC IDEAS & METAPHORS

1. **reconciliation of opposites into unity** (e.g. 'heaven' and 'earth'; light and darkness; male and female; spirit and matter, etc.)

2. **omphaloi** (literally: 'navels' [Greek]): centres, sources…

3. **triangles/3** (water; light; faith/love/hope; harmony; deflection of evil influence; stages of spiritual development – separation-fermentation-crumbling; birth-maturity-death)

4. **squares/4** ('terrestrial'/materiality; 4 directions [North, South, East, West]; 4 alchemical elements [earth, water, fire, air]; 4 stages of life [infancy, youth, maturity, old age])

5. **circles** ('heavenly'/ethereal; 'spirit'; unity and perfection; infinity; the wheel; the *kundalini* serpent)

B. USING ARCHETYPAL SHAPES, LITERALLY AND FIGURATIVELY

1. **Deploy people within the performance space according to these shapes**: (a) at their vertices and/or (b) along their perimeters. Shapes can overlay one another; they can be static or they can evolve into some other shapes.

2. **Stage activity can also 'trace out' shapes literally**. NB: bees' dances.

3. **Employ 'figurate number' patterns to connote geometric shapes abstractly**: (a) each performer independently thinks of, and works *strictly* with, some basic time-unit which they will mentally equate to '1'; (b) design 'triangular rhythms' $1 + 2 + 3 + …$ (or the reverse) or 'square rhythms' $1 + 3 + 5 + …$ (or the reverse); (c) use these as the bases for musicianly rhythmic-patterns and for the timing of various stage-actions (dance movements, etc.).

C. USING PROPORTIONS

1. **You will need**: basic arithmetical ability; a tape measure; a calculator; pen and paper; timekeeper.

2. **Use the human body itself as a data-source for ratios**! e.g. measure your height and calculate where your navel divides it. In theory (but rarely in practice), this should yield the "Golden Section" ratio $\phi = (\sqrt{5} - 1) \div 2 \approx 0.618$. I tried this myself: height 1966 mm; navel-to-floor 1193 mm; ensuing ratio 0.6068 (a 1.8% error relative to ϕ).

3. **Or instead, analyse the basic geometrical forms discussed above to obtain your elemental proportions**: see "Dimensiones Paradisi" page.

4. **Musicians**: given some overall duration for a piece, divide it into (sub)sections and/or rhythms according to such proportions.

5. **Dancers and Actors**: define the boundaries of your two-dimensional stage-space, and then partition it into sub-areas according to such proportions.

<div align="right">Dr Ian Shanahan</div>